

Fixed-Point Methods on Small-Signal Stability Analysis

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Abstract

In this paper we introduce the Diagonal Dominant Pole Spectrum Eigensolver (DDPSE), which is a fixed-point method that computes several eigenvalues of a matrix at a time. DDPSE is a slight modification of the Dominant Pole Spectrum Eigensolver (DPSE), that has being used in power system stability studies. We show that both methods have local quadratic convergence. Moreover, we present practical results obtained by both methods, from which we can see that those methods really compute dominant poles of a transfer function of the type $c^T(A - sI)^{-1}b$, where b and c are vectors, besides being also effective in finding low damped modes of a large scale power system.

1 Introduction

A power system can be described as a coupled system of differential and algebraic equations. The following system is obtained by linearizing the system at an operating point:

$$\begin{pmatrix} \dot{x} \\ 0 \end{pmatrix} = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix},$$

where J_1, J_2, J_3, J_4 are matrices, x is the vector of dynamical variables and y is the vector of algebraic ones. The matrix $J = \begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix}$ denotes the

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Jacobian matrix of the system. Since $y = -J_4^{-1}J_3x$, we have

$$\dot{x} = (J_1 - J_2J_4^{-1}J_3)x.$$

The matrix $A = J_1 - J_2J_4^{-1}J_3$ is called the state matrix of the system. For large scale power systems J is very sparse, while A is not in general. We can observe that, given a vector x , we can easily compute $z = (A - \lambda I)^{-1}x$ by solving the following algebraic system

$$\begin{pmatrix} J_1 & J_2 \\ J_3 & J_4 \end{pmatrix} \begin{pmatrix} z \\ w \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}.$$

From that we note that standard methods of eigenvalue calculation can be used in order to compute eigenvalues of A , even without explicitly calculating the state matrix.

Knowledge of rightmost eigenvalues of A is essential in the power system small-signal stability analysis. In the literature there are several papers that use classical methods to compute rightmost eigenvalues of a state matrix from the above calculation [1, 4, 5, 7, 10]. On the other hand, some authors prefer instead to deal with the generalized eigenvalue problem $Ju = \lambda Eu$, where

$$E = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix},$$

and I is the identity matrix. However, this approach requires a non-obvious strategy to control instability caused by the spurious eigenvalue at infinity, for instance, if you use generalized Möbius transforms [3, 6]. The landscape of small-signal stability analysis has changed a little when methods based on transfer functions, like DPA [8] and DPSE [9], have arisen in literature [9, 11]. The Dominant Pole Algorithm (DPA), which computes a single eigenvalue at a time, is actually a Newton's method according to a simple calculation shown in [2]. The Dominant Pole Spectrum Eigensolver (DPSE), which can be seen as a generalization of DPA in a certain way, is a fixed-point method that can compute several eigenvalues at a time. On the one hand, each step requires the solution of p linear systems if you want to calculate p eigenvalues. On the other hand, in power system stability studies, a suitable pre-ordering of the Jacobian matrix prevents large amounts of fill-in and thus its sparse LU factorization is done with lower computational complexity. Moreover, DPSE converges quadratically and a proof of its local quadratic convergence first appeared in [2]. Nevertheless, here we give an easier proof, which can

be seen in §2. We will also see that a slight modification of DPSE yields a new fixed-point method, the Diagonal Dominant-Pole Spectrum Eigensolver (DDPSE), that also has local quadratic convergence, as discussed in §3. In the last section, we compare results obtained from the implementation of those two methods regarding the time of computation. From these tests we verify that both methods really compute dominant poles of a transfer function of the type $c^T(A - sI)^{-1}b$, where b and c are vectors, besides being also effective in finding low damped modes of the system.

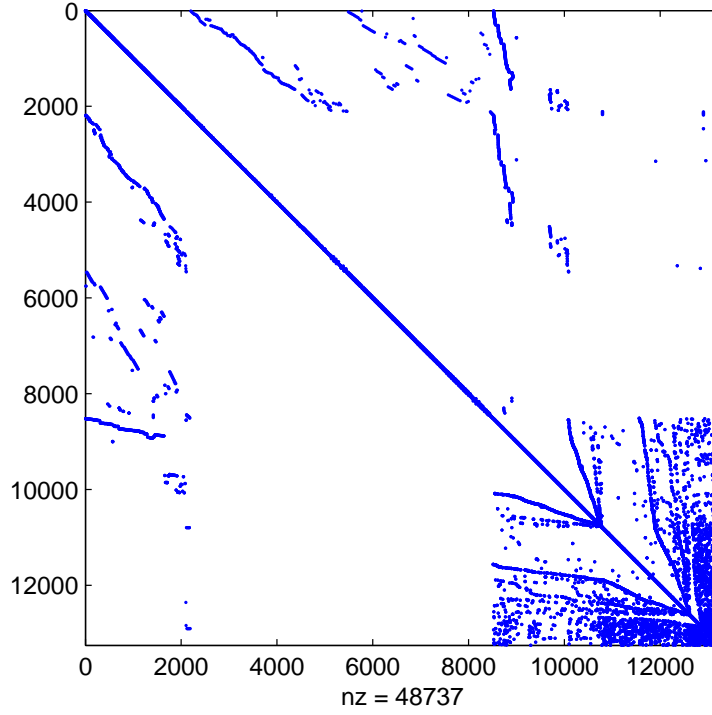


Figure 1: Sparse pattern of M (nz is the number of its nonzero entries)

2 DPSE

The motivation for the Dominant Pole Spectrum Eigensolver (DPSE) came from SISO dynamical systems (E, J, B, C, D) of the form

$$\begin{cases} E\dot{x}(t) &= Jx(t) + Bu(t) \\ y(t) &= C^T x(t) + Du(t) \end{cases}$$

where $J, E \in \mathbb{R}^{N \times N}$, $E = \text{diag}([1, \dots, 1, 0, \dots, 0])$; $x(t) \in \mathbb{R}^{N \times 1}$ is composed by dynamical and algebraic variables, $x_d(t)$ and $x_a(t)$, which are respectively associated with the unit and the null diagonal entries of E ; $B, C \in \mathbb{R}^{N \times 1}$, where $B^T = (B_d^T \ B_a^T)$ and $C^T = (C_d^T \ C_a^T)$; $u(t) \in \mathbb{R}$ is the input, $y(t) \in \mathbb{R}$ is the output, and $D \in \mathbb{R}$. The corresponding transfer function $h : \mathbb{C} \rightarrow \mathbb{C}$ is defined as

$$h(s) = C^T (sE - J)^{-1} B + D.$$

Suppose that there are n dynamical variables in the system. If $J_1 = J(1 : n, 1 : n)$, $J_2 = J(1 : n, n+1 : N)$, $J_3 = J(n+1 : N, 1 : n)$ and $J_4 = J(n+1 : N, n+1 : N)$, then

$$h(s) = c^T (sI - (J_1 - J_2 J_4^{-1} J_3))^{-1} b + d, \quad (1)$$

where $b = B_d - J_2 J_4^{-1} B_a$, $c = C_d - J_3^T J_4^{-T} C_a$ and $d = D - C_a^T J_4^{-1} B_a$. The matrix $A = J_1 - J_2 J_4^{-1} J_3$ is called the state matrix of the system. Note that, for any $\mu \notin \lambda(A)$ and for any $b \in \mathbb{C}^n$,

$$\begin{pmatrix} (A - \mu I)^{-1} b \\ 0 \end{pmatrix} = E (J - \mu E)^{-1} \begin{pmatrix} b \\ 0 \end{pmatrix}.$$

Suppose that $A \in \mathbb{R}^{n \times n}$ is diagonalizable, that is, $A = PDP^{-1}$, where P is an invertible matrix and D , a diagonal matrix. So, the spectrum of A , denoted by $\lambda(A)$, is the set of the diagonal entries of D . From now on we suppose that every eigenvalue of A is simple.

Let $b, c \in \mathbb{R}^n$ such that $c^T (A - sI)^{-1} b \neq 0$ for all $s \in \mathbb{C} - \lambda(A)$, and $c^T P e_k e_k^T P^{-1} b \neq 0$ for $k = 1 : n$. If $d = 0$, from Equation 1,

$$h(s) = \sum_{k=1}^n \frac{R_k}{d_{kk} - s},$$

where $R_k = c^T P e_k e_k^T P^{-1} b$, $k = 1 : n$.

Definition 1. A pole d_{kk} is called a dominant pole if it corresponds to a relatively large $m_k = \frac{|R_k|}{|Re(d_{kk})|}$. m_k is the measure for dominance of the pole d_{kk} .

Now, since

$$\frac{(A - sI)^{-1}b}{c^T(A - sI)^{-1}b} = \frac{Adj(A - sI)b}{c^T Adj(A - sI)b} \text{ and } \frac{(A^T - sI)^{-1}c}{c^T(A - sI)^{-1}b} = \frac{Adj(A^T - sI)c}{c^T Adj(A - sI)b},$$

we conclude that the functions $f : \mathbb{C} \rightarrow \mathbb{C}^n$ and $g : \mathbb{C} \rightarrow \mathbb{C}^n$ defined respectively by

$$f(s) = \begin{cases} \frac{(A-sI)^{-1}b}{c^T(A-sI)^{-1}b} & \text{for } s \in \mathbb{C} - \lambda(A); \\ \frac{Pe_j}{c^T Pe_j} & \text{for } s = d_{jj}, j = 1 : n. \end{cases} \quad (2)$$

and

$$g(s) = \begin{cases} \frac{(A^T-sI)^{-1}c}{c^T(A-sI)^{-1}b} & \text{for } s \in \mathbb{C} - \lambda(A); \\ \frac{P^{-T}e_j}{b^T P^{-T}e_j} & \text{for } s = d_{jj}, j = 1 : n. \end{cases} \quad (3)$$

are entire functions (bear in mind that any entry of the Classical Adjoint of $(A - sI)$ is a sum of products of its elements).

Let $S = (s_1 \dots s_p)^T \in \mathbb{C}^p$, $p \leq n$, and suppose that $X(S), Y(S) \in \mathbb{C}^{n \times p}$ are defined by $X(S)e_k = f(s_k)$ and $Y(S)e_k = g(s_k)$, for $k = 1 : p$, where e_1, \dots, e_n are the canonical vectors.

Lemma 1. Let $S_0 = (d_{k_1, k_1} \dots d_{k_p, k_p})$, where $d_{k_1, k_1} \dots d_{k_p, k_p}$ is a p -uple of distinct eigenvalues, $1 \leq k_1 < \dots < k_p \leq n$. Then, there is an open neighborhood \mathcal{O} of S_0 so that $Y(S)^T X(S)$ is invertible for any S belonging to \mathcal{O} .

Proof 1. The lemma follows because

$$Y(S_0)^T X(S_0) = \text{diag} \left(\left[\frac{1}{c^T P e_{k_1} e_{k_1}^T P^{-1} b}, \dots, \frac{1}{c^T P e_{k_p} e_{k_p}^T P^{-1} b} \right] \right), \quad (4)$$

that is, $Y(S_0)^T X(S_0)$ is invertible. \square

Let $F : \mathcal{O} \rightarrow \mathbb{C}^{n \times n}$ defined by $F(S) = (Y(S)^T X(S))^{-1} (Y(S)^T A X(S))$. We see that F is analytic. Since

$$F \left((d_{k_1, k_1} \dots d_{k_p, k_p})^T \right) = \text{diag} ([d_{k_1, k_1}, \dots, d_{k_p, k_p}]),$$

every eigenvalue of F is simple for S belonging to an open subset \mathcal{Z} of \mathcal{O} . Let $G : \mathcal{Z} \rightarrow \mathbb{C}^p$ be the function defined by $G(S) = (\lambda_1(F(S)) \dots \lambda_p(F(S)))$, where $\lambda_1(F(S)) < \dots < \lambda_p(F(S))$ are the eigenvalues of $F(S)$ (for some order on the complex numbers). Observe that $(d_{k_1, k_1} \dots d_{k_p, k_p})^T$ is a fixed point of G . On the other hand, if s_i is not an eigenvalue for $i = 1 : p$, we conclude that $F(S)$ is equal to

$$\text{diag}(S) + (Y(S)^T X(S))^{-1} e e^T \text{diag} \left(\left[\frac{1}{c^T (A - s_1 I)^{-1} b}, \dots, \frac{1}{c^T (A - s_p I)^{-1} b} \right] \right),$$

for $Y(S)b = e$, where $e = \text{ones}(p, 1)$. In order to calculate the derivative of G , we first see that

$$\frac{\partial F}{\partial s_i} (d_{k_1, k_1}, \dots, d_{k_p, k_p}) = e_i e_i^T - \frac{1}{c^T P e_{k_i} e_{k_i}^T P^{-1} b} v e_i^T,$$

where $v^T = (c^T P e_{k_1} e_{k_1}^T P^{-1} b \dots c^T P e_{k_p} e_{k_p}^T P^{-1} b)$. Therefore,

$$\frac{\partial \lambda_k}{\partial s_i} (d_{k_1, k_1}, \dots, d_{k_p, k_p}) = \left\langle \frac{\partial \lambda_k}{\partial a_{rs}} (F(d_{k_1, k_1}, \dots, d_{k_p, k_p})) \right\rangle, \frac{\partial F}{\partial s_i} (d_{k_1, k_1}, \dots, d_{k_p, k_p}) \rangle.$$

Notice that

$$\frac{\partial \lambda_k}{\partial a_{rs}} (F(d_{k_1, k_1}, \dots, d_{k_p, k_p})) = e_k e_k^T.$$

Hence,

$$\frac{\partial \lambda_k}{\partial s_i} (d_{k_1, k_1}, \dots, d_{k_p, k_p}) = e_i^T e_k - \frac{1}{c^T P e_{k_i} e_{k_i}^T P^{-1} b} e_i^T e_k e_k^T v = 0.$$

Definition 2 (DPSE). *The fixed-point iteration applied to the function*

$$G(S) = (\lambda_1(F(S)) \dots \lambda_p(F(S))),$$

where $F(S) = (Y(S)^T X(S))^{-1} (Y(S)^T A X(S))$, defines the Dominant Pole Spectrum Eigensolver.

With this definition, we have just proved the following proposition:

Proposition 2 (DPSE converges at least quadratically). *Let $\lambda_1, \dots, \lambda_p$ be p distinct eigenvalues of A . Then, there is a neighborhood V of $(\lambda_1 \dots \lambda_p)^T$ such that DPSE converges at least quadratically to $(\lambda_1 \dots \lambda_p)^T$ for any $S_0 \in V$.*

Remark 1. Suppose that DPSE has just calculated $S^{(r)} = \left(s_1^{(r)} \dots s_p^{(r)} \right)^T$ from $S^{(r-1)}$, and $X(S^{(r)})$ has not been computed yet. For any $k = 1 : p$, we have that

$$\begin{aligned} AX(S^{(r-1)})e_k &= \\ &= (A - s_k^{(r-1)}I + s_k^{(r-1)}I) \frac{(A - s_k^{(r-1)})^{-1}b}{c^T(A - s_k^{(r-1)})^{-1}b} = \\ &= \frac{b}{c^T(A - s_k^{(r-1)})^{-1}b} + s_k^{(r-1)}X(S^{(r-1)})e_k. \end{aligned}$$

Therefore, the relative error, $\|(A - s_k^{(r)})X(S^{(r-1)})e_k\|/\|X(S^{(r-1)})e_k\|$, becomes as follows:

$$\frac{\left\| \frac{b}{c^T(A - s_k^{(r-1)})^{-1}b} + (s_k^{(r-1)} - s_k^{(r)})X(S^{(r-1)})e_k \right\|}{\|X(S^{(r-1)})e_k\|}. \quad (5)$$

Notice that, if $s_k^{(r-1)}$ tends to an eigenvalue, then $c^T(A - s_k^{(r-1)})^{-1}b$ tends to zero, and so, $X(S^{(r-1)})e_k$ is an approximation of an associated eigenvector.

Remark 2. Suppose that, for some $j \in \{1, \dots, n\}$, d_{jj} is a converged eigenvalue at step r . Then, the corresponding right and left vectors, x_r and y_r , are such that

$$y_r^T x_r \approx \frac{e_j^T P^{-1}}{e_j^T P^{-1}b} \frac{P e_j}{c^T P e_j} = \frac{1}{c^T P e_j e_j^T P^{-1}b} = \frac{1}{R_j}.$$

3 DDPSE

If the fixed-point method is now applied to the diagonal of the matrix $F(S)$, then we have a variation of the DPSE, which will be called here as the Diagonal Dominant Pole Spectrum Eigensolver (DDPSE):

$$\begin{aligned} H \left((s_1 \dots s_p)^T \right) &\stackrel{\text{def}}{=} \text{diag} (F(s_1 \dots s_p)) = (s_1 \dots s_p)^T + \\ &+ \text{diag} \left(\left[\frac{1}{c^T(A - s_1 I)^{-1}b}, \dots, \frac{1}{c^T(A - s_p I)^{-1}b} \right] \right) (Y(S)^T X(S))^{-1} e \end{aligned}$$

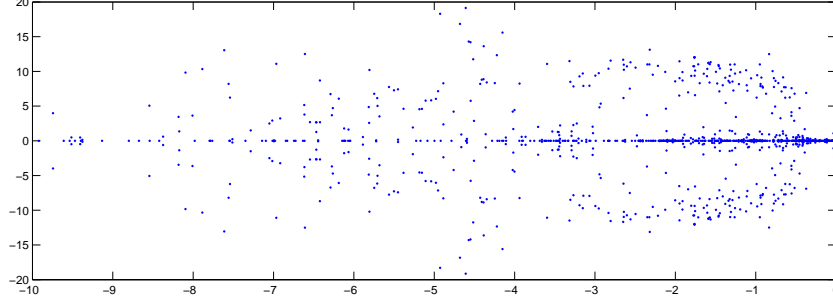


Figure 2: Partial spectrum of the state matrix

where $e = \text{ones}(p, 1)$ and $s_i \notin \lambda(A)$ for all $i = 1 : p$. Note that for $i = 1 : p$

$$\frac{1}{c^T(A - s_i I)^{-1}b} = \frac{\det(A - s_i I)}{c^T \text{Adj}(A - s_i I)b},$$

which is zero if s_i is an eigenvalue of A . So, we define

$$H((\lambda_1 \dots \lambda_p)^T) = (\lambda_1 \dots \lambda_p)^T$$

if $(\lambda_1 \dots \lambda_p)^T$ is a p -uple of distinct eigenvalues. Therefore, any p -uple of distinct eigenvalues is a fixed-point of H .

Proposition 3 (DDPSE converges at least quadratically). *Let $\lambda_1, \lambda_2, \dots, \lambda_p$ be p distinct eigenvalues of A . Then, there is a neighborhood V of $(\lambda_1 \dots \lambda_p)^T$ such that, given any $S_0 \in V$, DDPSE converges at least quadratically to $(\lambda_1 \dots, \lambda_p)^T$.*

Proof 2. Let $\lambda_1 = d_{k_1, k_1}, \dots, \lambda_p = d_{k_p, k_p}$. Then,

$$\frac{\partial H}{\partial s_i}((d_{k_1, k_1} \dots d_{k_p, k_p})) = e_i - \frac{1}{c^T P e_{k_i} e_{k_i}^T P^{-1} b} e_i e_i^T (Y(S)^T X(S))^{-1} e.$$

From 4, $(Y(S)^T X(S))^{-1} = \text{diag}\left(\left[c^T P e_{k_1} e_{k_1}^T P^{-1} b, \dots, c^T P e_{k_p} e_{k_p}^T P^{-1} b\right]\right)$. So,

$$\frac{\partial H}{\partial s_i}((d_{k_1, k_1} \dots d_{k_p, k_p})) = 0. \quad \square$$

k	ITER	CPU	k	ITER	CPU
16	21	21.9805	19	3	2.8080
4	22	23.0569	20	3	2.8080
17	22	23.0881	16	4	3.7128
20	23	23.9930	18	4	3.7128
3	24	24.7418	1	5	4.3992
7	24	24.7418	17	5	4.4304
8	24	24.7418	14	6	5.0700
12	24	24.7418	15	6	5.0700
19	24	24.7730	13	7	5.6940
5	25	25.3658	8	9	6.8016
6	25	25.3658	9	9	6.8016
9	25	25.3658	11	9	6.8016
11	25	25.3658	2	10	7.1448
14	25	25.3658	10	10	7.1760
18	25	25.3970	12	10	7.1760
15	26	25.8026	6	11	7.4880
2	28	26.4422	7	11	7.5192
1	33	27.5654	5	12	7.7532
13	34	27.7838	4	15	8.2993
10	35	27.8930	3	17	8.4709

Table 1: DDPSE versus DPSE

Re	Imag	m_k	Re	Imag	m_k
-0.6120	0.3587	12.40	-0.0335	-1.0787	760.11
-2.9957	-9.3891	0.57	-5.7475	6.7761	1.43
-4.5931	-0.2765	0.71	-1.2786	7.2546	1.97
-7.5416	-6.2291	1.07	-4.3601	0.9544	0.48
-1.2891	-8.5414	2.41	-4.1103	0.4801	1.04
-2.9445	-4.8214	6.85	-1.2936	1.4028	5.59
-4.0233	4.2124	2.61	-3.1928	9.2818	1.38
-2.9445	4.8214	6.85	-1.8415	6.9859	5.11
-9.7433	3.9765	0.14	-0.6120	0.3587	12.40
-2.2927	0.0000	2.92	-2.9445	4.8214	6.85
-5.8148	4.8704	1.36	-0.5776	6.2565	0.29
-3.1928	9.2818	1.38	-0.5550	4.1191	0.35
-0.0335	1.0787	760.11	-0.6786	7.1071	0.36
-0.5567	-3.6097	14.87	-1.2891	8.5414	2.41
-0.0335	-1.0787	760.15	-1.4790	8.2551	3.68
-0.9401	8.1931	0.37	-0.5208	2.8814	0.78
-1.2786	7.2546	1.96	-0.0335	1.0787	760.11
-5.7475	6.7761	1.43	-0.5567	3.6097	14.87
-5.5632	7.7510	1.22	-0.7584	4.9367	5.11
-1.4790	8.2551	3.68	-0.4548	4.7054	5.78

Table 2: Eigenvalues and their measure of relative dominance calculated by DDPSE and DPSE

4 Numerical results

Our test matrix J is sparse (density about 0.028%), of order $N = 13251$. The pencil $Jv = \lambda Ev$, where $E = \text{diag}(1, \dots, 1, 0, \dots, 0)$, corresponds to the problem $Ax = \lambda x$, where $A = J_1 - J_2 J_4^{-1} J_3$ is of order $n = 1664$. This is the Jacobian matrix that corresponds to a planning model of the Brazilian Interconnected Power System and that had already used for tests in [11]. In the tests with DPSE and DDPSE, we have used data which can be obtained from a specific transfer function. From this, $D = 0$, and the input vector $B = (B_d^T B_a^T)$ and the output vector $C = (C_d^T C_a^T)$ are as follows: $B(524) = B(1442) = 1$, $B(1884) = B(1918) = -1$, and the others entries of B are null; $C(11558) = 26.5721$, $C(11559) = -13.1127$, $C(12502) = -29.2954$, $C(12503) = 3.7609$ and these are all the non-zero elements of C . By using MATLAB, $C_a^T J_4^{-1} B_a = 0$. Note that, from Equation 1, for any $s \notin \lambda(A)$, $C^T (J - sE)^{-1} B = c^T (A - sI)^{-1} b$, where $b = B_d - J_2 J_4^{-1} B_a$, $c = C_d - J_3^T J_4^{-T} C_a$.

Let $s^{(k)} = (s_1^{(k)} \dots s_p^{(k)})^T \in \mathbb{C}^p$. Let $X = X^{(k)}$ and $Y = Y^{(k)}$ be two matrices $N \times p$, such that for $j = 1 : p$ $X(:, j) = (J - s_j^{(k)} E)^{-1} B / C^T (J - s_j^{(k)} E)^{-1} B$ and $Y(:, j) = (J^T - s_j^{(k)} E)^{-1} C / C^T (J - s_j^{(k)} E)^{-1} B$. Let $V = V^{(k)}$ and $W = W^{(k)}$ be two matrices $n \times p$ such that, for $j = 1 : p$,

$$V(:, j) = (A - s_j^{(k)} I)^{-1} b / c^T (A - s_j^{(k)} I)^{-1} b$$

and

$$W(:, j) = (A^T - s_j^{(k)} I)^{-1} c / c^T (A - s_j^{(k)} I)^{-1} b.$$

So, we have

$$W^T V = Y^T E X,$$

and for $j = 1 : p$

$$\begin{aligned} W^T A V e_j &= W^T (A - s_j^{(k)} I + s_j^{(k)} I) (A - s_j^{(k)} I)^{-1} b / c^T (A - s_j^{(k)} I)^{-1} b = \\ &= W^T b / c^T (A - s_j^{(k)} I)^{-1} b + s_j^{(k)} W^T V e_j = e / c^T (A - s_j^{(k)} I)^{-1} b + s_j^{(k)} W^T V e_j, \end{aligned}$$

where $e = \text{ones}(p, 1)$. Therefore,

$$F = (W^T V)^{-1} W^T A V = (W^T V)^{-1} e v^T + S,$$

where $S = S^{(k)} = \text{diag}(s^{(k)})$, and

$$v^T = (v^{(k)})^T = \left(1 / c^T (A - s_1^{(k)} I)^{-1} b \dots 1 / c^T (A - s_p^{(k)} I)^{-1} b \right).$$

Hence, in order to use DPSE with A , b and c , we can carry out all the computation with J , B and C without explicitly computing A , b and c .

We have specified a relative error tolerance of 10^{-5} to both right and left vectors. Here we have used

$$\frac{\|(J - s_j^{(k)}E)X^{(k-1)}e_j\|}{\|X^{(k-1)}e_j\|} \text{ and } \frac{\|(J^T - s_j^{(k)}E)Y^{(k-1)}e_j\|}{\|Y^{(k-1)}e_j\|}$$

as the relative errors instead of using the equivalent formulae obtained from Equation 5, for we do not want to compute b explicitly. Convergence is achieved when both the convergence criteria are satisfied. Suppose we have just obtained the first converged value $\lambda_1 = s_1^{(k)}$. Then, we save their corresponding right and left generalized eigenvectors, $X^{(k-1)}e_1$ and $Y^{(k-1)}e_1$, to the respective columns of $X^{(r)}$ and $Y^{(r)}$ for $r \geq k$. Notice that the respective columns of $V^{(r)}$ and $W^{(r)}$ are formed by the dynamical variables of $X^{(k-1)}e_1$ and $Y^{(k-1)}e_1$, respectively. That eigenvalue can be deflated from the problem by this procedure. To see that, suppose $Ve_2 \approx a_1v_1 + a_2v_2$, where $v_1 \approx V^{(k-1)}e_1$ and v_2 is an eigenvector corresponding to λ_2 . Hence,

$$AVe_2 \approx a_1\lambda_1v_1 + a_2\lambda_2v_2 \approx a_1(\lambda_1 - \lambda_2)Ve_1 + \lambda_2Ve_2,$$

and so,

$$(W^TV)^{-1}W^TAVe_2 \approx a_1(\lambda_1 - \lambda_2)e_1 + \lambda_2e_2.$$

Thus,

$$F = \begin{pmatrix} \lambda_1 & \times & \times & \cdots & \times \\ 0 & \lambda_2 & \times & \cdots & \times \\ 0 & 0 & \times & \cdots & \times \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \times & \cdots & \times \end{pmatrix}.$$

In Table 1, for each eigenvalue calculated by DDPSE, or by DPSE, we list the CPU time (in seconds) and the number of iterations required for convergence, together with the corresponding measure m_k for dominance of a pole that was computed by the respective method. For the tests we have started DPSE and DDPSE by choosing the same 20 initial shifts given in the left complex semi-plane: $\mu_k = k * (-1/20 + i/2)$, $k = 1 : 20$.

The tests have been performed in the MATLAB R2011b 64 bits at a HP Compaq 6000 Pro, with processor Intel Core 2 Duo E8400 3.00 GHz.

4.1 Conclusions

In the tests, DPSE has shown that it is more stable than DDPSE. For instance, both algorithms started with the same 20 complex numbers in the upper-half plane. However, while DPSE converged to 19 eigenvalues still located in the upper half-plane, DDPSE converged to only 13 eigenvalues with positive imaginary part. On the other hand, both converged to $-0.0335 \pm 1.0787i$, which are the most dominant poles of the system. The DDPSE algorithm typically converges more slowly than the DPSE in total computer time, and we see that in Table 1. Note that the eigenvalues of A are clustered around zero, according to Figure 2, and even so both algorithms converged to dominant poles.

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